

Electromagnetic scattering on extraterrestrial D3-brane spikes¹

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Abstract

We consider scattering of electromagnetic plane waves on a D3-brane spike which emanates normal to D3-brane in the extra space direction. Previous studies show that this D3-brane spike soliton of the Dirac-Born-Infeld Lagrangian can be interpreted as a F/D-string attached to a D3-brane and that Polchinski's open string boundary conditions realize here dynamically. Now we are interested in studying physical effects which can be produced by this extraterrestrial spike attached to our D3-brane world. We have observed that extraterrestrial spike sucks almost all electromagnetic radiation and therefore acts as a black hole. This is because absorption cross section tends to a constant at low energy limit, instead, the scattered part of the radiation on a D3-brane tends to zero demonstrating non-Thompson behaviour, universal for the charged objects in electrodynamics. Comparing these two scattering cross sections we can extract effective mass of the spike: it depends on incident radiation frequency and tends to infinity in the low energy limit.

¹Dedicated to the memory of Professor Guren Sahakian
<http://server.physdep.r.am/gsahak/>

1 Introduction

The Dp-brane is a dynamical extended object which can fluctuate in shape and position and these fluctuations are described by open string massless modes [1, 2, 3]. For a single Dp-brane they are massless vector and spinor states of ten-dimensional $N = 1$ supersymmetric $U(1)$ gauge theory. The massless field $A_\mu(x^\nu)$, $\mu, \nu = 0, \dots, p$ propagates as gauge bosons on the p-brane worldvolume, while the other components of the vector potential $A_{p+1}(x^\nu), \dots, A_9(x^\nu)$ describe the transverse deviations of the Dp-brane $x_{p+1} \equiv \phi_{p+1}, \dots, x_9 \equiv \phi_9$. The low energy effective action for these fields is the Dirac-Born-Infeld action [3, 4, 5].

Callan and Maldacena [7, 8, 9] showed that the Dirac-Born-Infeld action supports solitonic configurations describing F(fundamental)-strings and D(Dirichlet)-strings growing out of the original D3-brane. These configurations have nonzero world-volume gauge field and transverse scalar field excited. The gauge field describes a point electric or magnetic charge arising from the end-point of the attached string and the scalar field represents a deformation of the D3-brane in the form of infinite spike (see Figure 1). This is extraterrestrial spike protruding from our D3-brane world into extra space direction and it is interesting to study physical effects which are produced by this extraterrestrial spike growing out of our universe. In particular it is interesting to consider electromagnetic scattering on these extraterrestrial spikes and to study physical effects which they produce on a D3-brane world.

The spike soliton, F-string, which satisfies 1/2-BPS conditions of the $D = 4$ $N = 4$ supersymmetric electrodynamics is [7],

$$\vec{E} = \frac{e^2}{r^2} \vec{e}_r, \quad \vec{\partial} x_9 = \frac{e^2}{r^2} \vec{e}_r, \quad x_9 = -\frac{e^2}{r}, \quad (1)$$

where $2e^2 = g_s$ is a unit charge. Here the scalar field represents a geometrical spike, and the electric field insures that the string carries uniform NS charge along it. It was demonstrated in [7] that the infinite electrostatic energy of the point charge can be reinterpreted as being due to the infinite length of the attached string. The energy per unit length comes from the electric field and corresponds exactly to the fundamental string tension. As it was shown in [7, 10, 11] small fluctuations which are normal to both the string and the brane are mostly reflected back with a *phase shift* $\rightarrow \pi$ thus realizing dynamically Polchinski's open string Dirichlet boundary condition ². In [13, 14, 15] it was also demonstrated that P-wave excitations (j=1) which are coming down the string with a polarization along a direction parallel to the brane are almost completely reflected just as in the case of all-normal excitations, but now the end of the string moves freely on the 3-brane. This corresponds to the reflection of the geometrical fluctuation with a *phase shift* $\rightarrow 0$, thus realizing dynamically Polchinski's open string Neumann boundary condition.

In [13] it was also observed the electromagnetic dipole radiation which escapes to infinity from the place where the string is attached to the D3-brane. It was shown that in the low energy limit $R\omega \rightarrow 0$ ($R^2 = e^2(2\pi\alpha')$), i.e. for wavelengths much larger than the string scale a small fraction $\sim \omega^4$ of the energy escapes to infinity in the form of electromagnetic dipole radiation (j=1). The physical interpretation was that a string

²See also [12] for a supergravity treatment of this problem.

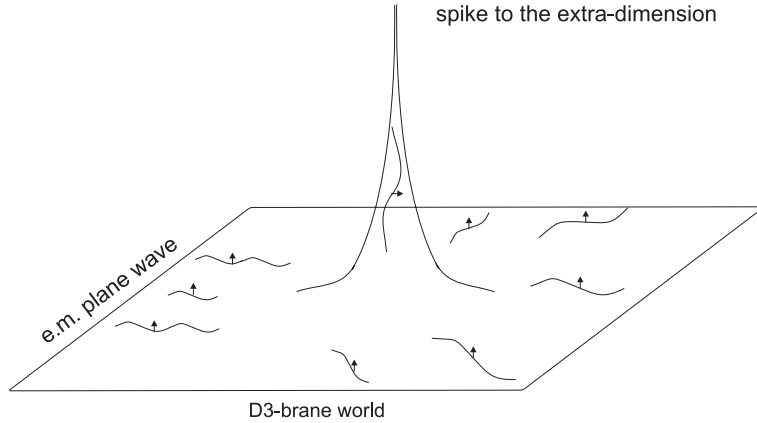


Figure 1: This configuration describes D3-brane spike growing out of the original D3-brane and can be interpreted as a string attached to a D3-brane. We consider scattering of electromagnetic plane waves on a D3-brane spike which emanate normal to D3-brane in the extra space direction. The extraterrestrial spike sucks almost all electromagnetic radiation, because absorption cross section tends to a constant at low energy limit, instead, the scattered part of radiation on a D3-brane tends to zero demonstrating non-Thompson behaviour, universal for the charged objects in electrodynamics.

attached to the D3-brane manifests itself as an electric charge, and waves on the string cause the end point of the string to freely oscillate and produce electromagnetic dipole radiation in the asymptotic outer region of the D3-brane. Thus dynamics of the spike, as probed through small fluctuations, agree with existing string behaviour.

The propagation of higher angular momentum modes has been analyzed in [16], where authors came to the conclusion that they also propagate along the spike, which means that not only dipole radiation with $j = 1$ but also higher modes travel along the spike and hence demonstrate that the spike remains effectively $3 + 1$ dimensional. This fact causes problem with a string interpretation of the spike soliton because only $j = 1$ mode should propagate on a fundamental string. A proposal to solve this puzzle was suggested in [17, 18] where authors consider noncommutative solution describing N coincident D-strings attached to a D3-brane in non-Abelian theory [20, 22, 23, 24]. This solution accurately describes the physics at the core of the spike, very far from the D3-brane, and coincides with the above spike soliton in Abelian world-volume theory in the large N limit. The analysis shows that on noncommutative spike the mode spectrum is truncated at $l_{max} = N - 1$ [17, 18].

In this article we are interested in studying physical effects which are produced on D3-brane by a spike which emanates normal to D3-brane world in the extra space direction. Our aim is to consider scattering of electromagnetic plane waves on this D3-brane spike. Specifically we are interested in studying reflected radiation produced by extraterrestrial spike attached to our world. We have observed that extraterrestrial spike sucks almost all electromagnetic radiation and therefore acts as a black hole. This is because absorption cross section for $j = 1$ tends to a *constant* at low energy limit ($R\omega \rightarrow 0$)

$$\sigma_1^{abs} \sim R^2 \rightarrow Const, \quad \sigma_j^{abs} \sim R^2 (R\omega)^{4j-4} \rightarrow 0, \quad j = 2, 3, \dots$$

This behaviour is appealing for a string interpretation of the spike soliton because it is a constant for $j = 1$ and tends to zero for higher $j > 1$, therefore the propagation of $j = 1$

mode is indeed distinctive³. Instead, the scattered part of radiation on a D3-brane *tends to zero* for all $j = 1, 2, \dots$

$$\sigma_j^{scat} \sim R^2(R\omega)^{8j-6} \rightarrow 0, \quad j = 1, 2, 3, \dots$$

demonstrating non-Thompson behaviour for $j = 1$, universal for charged objects in electrodynamics. Comparing these two scattering cross sections we can extract the effective mass of the spike

$$M_{eff}^2 = \frac{e^4}{R^2(R\omega)^2}.$$

It is finite for nonzero $R\omega$, demonstrating that dynamical mass involved in the process is finite and is of order of the wave length $1/\omega$. In the low energy limit it tends to infinity demonstrating that "static" mass of the spike is infinite.

2 The Lagrangian and the equations

The nonlinear Dirac-Born-Infeld Lagrangian which contains both electric and magnetic fields, plus the scalar $x_9 \equiv \phi$ is [13, 15]

$$L = - \int d^4x \sqrt{Det} \quad \text{where} \quad Det = 1 + \vec{B}^2 - \vec{E}^2 - (\vec{E} \cdot \vec{B})^2 - (\partial_0\phi)^2(1 + \vec{B}^2) + (\vec{\partial}\phi)^2 + (\vec{B} \cdot \vec{\partial}\phi)^2 - (\vec{E} \times \vec{\partial}\phi)^2 + 2\partial_0\phi(\vec{B}[\vec{\partial}\phi \times \vec{E}]). \quad (2)$$

and can also be written in a covariant form

$$Det = 1 + \frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \partial_\mu\phi\partial^\mu\phi - \left(\frac{1}{8}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}\right)^2 + \left(\frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\nu\lambda}\partial_\rho\phi\right)^2.$$

The full set of field equations can be obtained by variation

$$\partial_\mu \left\{ \frac{F^{\mu\nu}(1 - \partial_\rho\phi\partial^\rho\phi) - \frac{1}{32}(F_{\lambda\rho}\tilde{F}^{\lambda\rho})\tilde{F}^{\mu\nu} + F^{\mu\rho}\partial_\rho\phi\partial^\nu\phi + \partial^\mu\phi\partial_\rho\phi F^{\rho\nu}}{Det^{1/2}} \right\} = 0, \quad (3)$$

$$\partial_\mu \left\{ \frac{\partial^\mu\phi(1 - F_{\lambda\rho}F^{\lambda\rho}) + 2F^{\mu\nu}F_{\nu\lambda}\partial^\lambda\phi}{Det^{1/2}} \right\} = 0, \quad \epsilon^{\mu\nu\lambda\rho}\partial_\nu F_{\lambda\rho} = 0, \quad (4)$$

where $\tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}$. It is also useful to have equations of non-linear scalar electrodynamics in a component form

$$\begin{aligned} & \vec{\nabla} \times \left\{ \frac{\vec{B}(1 - (\partial_0\phi)^2) - \vec{E}(\vec{E}\vec{B}) + \vec{\nabla}\phi(\vec{B}\vec{\nabla}\phi) + \partial_0\phi(\vec{\nabla} \times \vec{E})}{Det^{1/2}} \right\} \\ &= \frac{\partial}{\partial t} \left\{ \frac{\vec{E}(1 + (\vec{\nabla}\phi)^2) + \vec{B}(\vec{E}\vec{B}) - \vec{\nabla}\phi(\vec{E}\vec{\nabla}\phi) + \partial_0\phi(\vec{\nabla} \times \vec{B})}{Det^{1/2}} \right\} + 4\pi\vec{j} \end{aligned} \quad (5)$$

together with the corresponding Gauss law

$$\vec{\nabla} \left\{ \frac{\vec{E}(1 + (\vec{\nabla}\phi)^2) + \vec{B}(\vec{E}\vec{B}) - \vec{\nabla}\phi(\vec{E}\vec{\nabla}\phi) + \partial_0\phi(\vec{\nabla} \times \vec{B})}{Det^{1/2}} \right\} = 4\pi\rho. \quad (6)$$

³The apparent difference between this result and [16] is that computing cross sections we have divided the scattered flux of radiation by the incident flux of electromagnetic field, which is proportional to ω^2 .

The rest two equations are the same as in Maxwell's theory $\vec{\nabla}\vec{B}=0$, $\vec{\nabla}\times\vec{E}=\frac{\partial\vec{B}}{\partial t}$ and the equation for the scalar field is:

$$\frac{\partial}{\partial t}\left\{\frac{\partial_0\phi(1+\vec{B}^2)-\vec{B}(\vec{\nabla}\phi\times\vec{E})}{Det^{1/2}}\right\}-\vec{\nabla}\left\{\frac{\vec{\nabla}\phi(1-\vec{E}^2)+\vec{E}(\vec{E}\vec{\nabla}\phi)+\vec{B}(\vec{B}\vec{\nabla}\phi)+\partial_0\phi(\vec{E}\times\vec{B})}{Det^{1/2}}\right\}=0. \quad (7)$$

We have to consider now small fluctuations around the background field configuration of spike soliton (1) :

$$\vec{E}=\vec{E}_0+\delta\vec{E}, \quad \vec{B}=\delta\vec{B}, \quad \phi=\phi_0+\eta.$$

Then keeping only terms in the Det which are linear and quadratic in the fluctuation one can get the resulting quadratic Lagrangian

$$2L_q = \delta\vec{E}^2(1+(\vec{\partial}\phi)^2) - \delta\vec{B}^2 + (\partial_0\eta)^2 - (\vec{\partial}\eta)^2(1-\vec{E}_0^2) + \vec{E}_0^2(\vec{\partial}\eta\cdot\delta\vec{E}). \quad (8)$$

Let us introduce the gauge potential for the fluctuation part of the e.m. field as (A_0, \vec{A}) and substitute the values of the background fields from (1)

$$2L_q = (\partial_0\vec{A}-\vec{\partial}A_0)^2(1+\frac{R^4}{r^4}) - (\vec{\nabla}\times\vec{A})^2 + (\partial_0\eta)^2 - (\vec{\partial}\eta)^2(1-\frac{R^4}{r^4}) + \frac{R^4}{r^4}(\partial_0\vec{A}-\vec{\partial}A_0)\cdot\vec{\partial}\eta, \quad (9)$$

where we restore the dimensions of the fields and $R^2 = e^2(2\pi\alpha')$. The equations that follow from this Lagrangian contain dynamical equations for the vector potential and for the scalar field, and a separate equation which represents a constraint. These equations in the Lorenz gauge $\vec{\partial}\cdot\vec{A} = \partial_0A_0$ are

$$-\partial_0^2\vec{A}(1+\frac{R^4}{r^4}) + \Delta\vec{A} + \frac{R^4}{r^4}\vec{\partial}\partial_0(A_0+\eta) = 0 \quad (10)$$

$$-\partial_0^2A_0 + \Delta A_0 + \vec{\partial}\frac{R^4}{r^4}\vec{\partial}(A_0+\eta) - \vec{\partial}\frac{R^4}{r^4}\partial_0\vec{A} = 0 \quad (11)$$

$$-\partial_0^2\eta + \Delta\eta - \vec{\partial}\frac{R^4}{r^4}\vec{\partial}(A_0+\eta) + \vec{\partial}\frac{R^4}{r^4}\partial_0\vec{A} = 0 \quad (12)$$

Equation (11) is a constraint: the time derivative of the *lhs* is zero, as can be shown using the equation of motion (10). One can choose $A_0 = -\eta$ which is compatible with the field equations because the second and the third equations imply that $A_0 + \eta$ obeys the free wave equation. With this condition the equations (11) and (12) become the same, and the first equation is also simplified [13]:

$$\Delta\vec{A} - (1+\frac{R^4}{r^4})\ddot{\vec{A}} = 0, \quad (13)$$

$$\Delta\eta - \ddot{\eta} + \vec{\partial}(\frac{R^4}{r^4}\dot{\vec{A}}) = 0, \quad (14)$$

where $\dot{\vec{A}}$ denotes time derivative. This should be understood to imply that once we obtain a solution, A_0 is determined from η , but in addition we are now obliged to respect the gauge condition which goes over to $\vec{\partial}\vec{A} = -\dot{\eta}$.

In the next section we shall consider the scattering of plane waves on a spike soliton (1). It is always possible to choose the plane wave to be a 1/2-BPS configuration. For that one should take the plane wave solution of the equations (3),(4) in the form:

$$A_\mu = e_\mu \phi_0 e^{ikx}, \quad \phi = -\phi_0 e^{ikx}, \quad (15)$$

where $e_\pm^\mu = (\omega, \vec{e}_\pm + \vec{k})$, $\vec{e}_\pm * \vec{k} = 0$, and in addition we have $\partial_\mu A^\mu = 0$, $A_0 + \phi = 0$.

3 Electromagnetic scattering on a extraterrestrial spike

Below we shall consider stationary scattering with definite energy (frequency ω)

$$\begin{aligned} \Delta \vec{A} + \omega^2 \left(1 + \frac{R^4}{r^4}\right) \vec{A} &= 0, \\ \Delta \eta + \omega^2 \eta - \vec{\nabla} \left(\frac{i\omega R^4}{r^4} \vec{A} \right) &= 0, \\ \vec{\nabla} \vec{A} &= i\omega \eta. \end{aligned} \quad (16)$$

Let us consider following expansion for the vector potential

$$\vec{A} = \vec{e} \sum_l Y_{l0}(\theta) \zeta_l,$$

where \vec{e} is the constant polarization vector and ζ_l are the partial waves. From (16) one can get equation which they should satisfy

$$\partial_r^2 \zeta_l + \frac{2}{r} \partial_r \zeta_l + \left[\omega^2 \left(1 + \frac{R^4}{r^4}\right) - \frac{l(l+1)}{r^2} \right] \zeta_l = 0. \quad (17)$$

With the substitution similar to the one used in [25]

$$r = R e^{-z} \quad \zeta_l = e^{-z/2} \tilde{\zeta}_l \quad (18)$$

it can be transformed into the form

$$[\partial_z^2 + 2(R\omega)^2 e^{2z} - (l+1/2)^2] \tilde{\zeta}_l = 0. \quad (19)$$

This is the well known Mathieu's equation. The known mathematics of Mathieu's equation [26, 27, 28] allows to calculate a systematic expansion of solution in all orders in $(R\omega/2)$. The formulas needed for our purposes are summarized in [29]. There are two asymptotic regions in our case: the first one $z \gg \ln(R\omega)$ (I) where we have

$$[\partial_z^2 + (R\omega)^2 e^{2z} - (l+1/2)^2] \tilde{\zeta}_l = 0 \quad (20)$$

and the second one $z \ll -\ln(R\omega)$ (II) where

$$[\partial_z^2 + (R\omega)^2 e^{-2z} - (l+1/2)^2] \tilde{\zeta}_l = 0. \quad (21)$$

The solutions in these two regions are known functions

$$\tilde{\zeta}_l^I = H_{l+1/2}^{(1)}(R\omega e^z), \quad \tilde{\zeta}_l^{II} = A_\infty J_{l+1/2}(R\omega e^{-z}) + B_\infty N_{l+1/2}(R\omega e^{-z}). \quad (22)$$

The two regions overlap in the interval $\ln(R\omega) \ll z \ll -\ln(R\omega)$ if $R\omega \ll 1$. In this interval we can use the following expansion of the functions:

$$\begin{aligned} H_{l+1/2}^{(1)}(\xi) &\simeq (\xi/2)^{l+1/2} \frac{1}{\Gamma(l+3/2)} + i(\xi/2)^{-l-1/2} \frac{(-1)^{l+1}}{\Gamma(-l+1/2)}, \\ J_{l+1/2}(\xi) &\simeq (\xi/2)^{l+1/2} \frac{1}{\Gamma(l+3/2)}, \quad N_{l+1/2}(\xi) \simeq (\xi/2)^{-l-1/2} \frac{(-1)^{l+1}}{\Gamma(-l+1/2)} \end{aligned}$$

and thus to impose matching condition

$$\begin{aligned} \left(\frac{R\omega}{2}\right)^{l+1/2} \frac{e^{(l+1/2)z}}{\Gamma(l+3/2)} + i(-1)^{l+1} \left(\frac{R\omega}{2}\right)^{-l-1/2} \frac{e^{-(l+1/2)z}}{\Gamma(-l+1/2)} &\simeq \\ A_\infty \left(\frac{R\omega}{2}\right)^{l+1/2} \frac{e^{-(l+1/2)z}}{\Gamma(l+3/2)} + B_\infty (-1)^{l+1} \left(\frac{R\omega}{2}\right)^{-l-1/2} \frac{e^{(l+1/2)z}}{\Gamma(-l+1/2)}. \end{aligned} \quad (23)$$

From this one can find coefficients A_∞ and B_∞

$$A_\infty = i(-1)^{l+1} \left(\frac{R\omega}{2}\right)^{-2l-1} \frac{\Gamma(l+3/2)}{\Gamma(-l+1/2)}, \quad B_\infty = (-1)^{l+1} \left(\frac{R\omega}{2}\right)^{2l+1} \frac{\Gamma(-l+1/2)}{\Gamma(l+3/2)}. \quad (24)$$

Using the asymptotic expansion for $\xi \rightarrow \infty$

$$\begin{aligned} H_\nu^{(1)}(\xi) &\simeq \sqrt{\frac{2}{\pi\xi}} \exp\left(\xi - \frac{\pi}{2}\nu - \frac{\pi}{4}\right), \\ J_\nu(\xi) &\simeq \sqrt{\frac{2}{\pi\xi}} \cos\left(\xi - \frac{\pi}{2}\nu - \frac{\pi}{4}\right), \quad N_\nu(\xi) \simeq \sqrt{\frac{2}{\pi\xi}} \sin\left(\xi - \frac{\pi}{2}\nu - \frac{\pi}{4}\right) \end{aligned}$$

and the formulas (19) we can recover the asymptotic behaviour of the solution at the origin where the spike is attached to the D3-brane

$$r \rightarrow 0 \quad \zeta_l \simeq \sqrt{\frac{2}{\pi R\omega}} e^{-i\pi(l+1)/2} e^{\frac{i\omega R^2}{r}}, \quad (25)$$

and at infinity from the origin

$$\begin{aligned} r \rightarrow \infty \quad \zeta_l &\simeq \frac{R}{2r} \sqrt{\frac{2}{\pi R\omega}} \{e^{-i\pi(l+1)/2} (A_\infty - iB_\infty) e^{i\omega r} + e^{i\pi(l+1)/2} (A_\infty + iB_\infty) e^{-i\omega r}\} = \\ &= \frac{R}{2r} \sqrt{\frac{2}{\pi R\omega}} e^{i\pi(l+1)/2} (A_\infty + iB_\infty) [e^{-i\omega r} + R_l e^{i\omega r}]. \end{aligned} \quad (26)$$

The first asymptotic (25) describes absorption waves which penetrate through the potential barrier and propagate along the spike, the second one (26) describes the scattered radiation. Thus the transmission and reflection amplitudes are equal to:

$$T_l = \frac{e^{-i\pi(l+1)/2}}{\frac{R}{2} e^{+i\pi(l+1)/2} (A_\infty + iB_\infty)} = -i \left(\frac{2}{R}\right) \left(\frac{R\omega}{2}\right)^{2l+1} \frac{\Gamma(-l+1/2)}{\Gamma(l+3/2)}, \quad (27)$$

$$R_l = \frac{e^{-i\pi(l+1)/2} (A_\infty - iB_\infty)}{e^{+i\pi(l+1)/2} (A_\infty + iB_\infty)} = (-1)^{l+1} [1 - 2 \left(\frac{R\omega}{2}\right)^{4l+2} \left(\frac{\Gamma(-l+1/2)}{\Gamma(l+3/2)}\right)^2], \quad (28)$$

and for the lower partial waves they are:

$$\begin{aligned} T_0 &= -2i\omega, & R_0 &= -1 + 2(R\omega)^2; \\ T_1 &= \frac{2i}{3}\omega(R\omega)^2, & R_1 &= 1 - \frac{2}{9}(R\omega)^6; \\ T_2 &= -\frac{2i}{45}\omega(R\omega)^4, & R_2 &= -1 + \frac{2}{45^2}(R\omega)^{10}. \end{aligned} \quad (29)$$

For the large partial waves they behave as:

$$T_l \sim \omega(R\omega)^{2l}, \quad R_l - (-1)^{l+1} \sim (R\omega)^{4l+2} \quad (30)$$

4 Cross Sections

The asymptotic behaviour of the wave function can be represented in the form:

$$\begin{aligned} \vec{A} &= \vec{e} \left(e^{ikz} + \frac{f(\theta)}{r} e^{ikr} + g(\theta) e^{ikR^2/r} \right) = \\ &= \vec{e} \sum_{l=0}^{\infty} \frac{\sqrt{4\pi(2l+1)}}{2ik(-1)^{l+1}} Y_{l0}(\theta) \left[\frac{1}{r} e^{-ikr} + (-1)^{l+1} \frac{1}{r} (1 + 2ikf_l) e^{ikr} + (-1)^{l+1} 2ikg_l e^{ikR^2/r} \right], \end{aligned} \quad (31)$$

where

$$f(\theta) = \sum_{l=0}^{\infty} \sqrt{4\pi(2l+1)} Y_{l0}(\theta) f_l$$

and comparing this with the asymptotic solution (25), (26) and (27), (28) we can find that $R_l = (-1)^{l+1}(1 + 2i\omega f_l)$ and $T_l = (-1)^{l+1}2i\omega g_l$ and therefore:

$$f_l = \frac{i}{\omega} \left(\frac{R\omega}{2} \right)^{4l+2} \left(\frac{\Gamma(-l+1/2)}{\Gamma(l+3/2)} \right)^2, \quad g_l = (-1)^l \frac{1}{R\omega} \left(\frac{R\omega}{2} \right)^{2l+1} \frac{\Gamma(-l+1/2)}{\Gamma(l+3/2)}. \quad (32)$$

In particular for f_0 and g_0 they are equal to

$$f_0 = iR(R\omega), \quad f_1 = \frac{i}{9}R(R\omega)^5, \quad g_0 = 1, \quad g_1 = \frac{1}{3}(R\omega)^2. \quad (33)$$

If dI is the energy radiated by the system into spherical angle $d\Omega$ and incident electromagnetic plane wave has Pointing vector \vec{S} then the cross section is equal to $d\sigma = \frac{dI}{S}$. The Pointing vector \vec{S} for the incident plane wave $\vec{e} e^{ikz}$ is equal to $\vec{S}_{in} = \frac{1}{4\pi} B_{in}^2 \vec{e}_z = \frac{\omega^2}{4\pi} \vec{e}_z$ and for the scattered radiation $\vec{e} \frac{f(\theta)}{r} e^{ikr}$ and absorption $\vec{e} g(\theta) e^{ikR^2/r}$ waves it is

$$\vec{S}_{scat} = \vec{n} \frac{1}{4\pi} B_{scat}^2, \quad \vec{S}_{abs} = -\vec{n} \frac{1}{4\pi} B_{abs}^2, \quad \vec{n} = \frac{\vec{k}}{k}.$$

Thus we have to find \vec{B}_{scat} and \vec{B}_{abs}

$$\vec{B}_{scat} = \vec{\nabla} \times \vec{A} = i[\vec{k} \times \vec{e}] \frac{f(\theta)}{r} e^{ikr}, \quad \vec{B}_{abs} = -i[\vec{k} \times \vec{e}] g(\theta) \frac{R^2}{r^2} e^{ikR^2/r},$$

and then the \vec{S}_{scat} and the \vec{S}_{abs}

$$\vec{S}_{scat} = \vec{n} \frac{\omega^2}{4\pi} |\vec{n} \times \vec{e}|^2 \frac{|f(\theta)|^2}{r^2}, \quad \vec{S}_{abs} = -\vec{n} \frac{\omega^2}{4\pi} |\vec{n} \times \vec{e}|^2 |g(\theta)|^2 \frac{R^4}{r^4}$$

and the radiated energy into the spherical angle $d\Omega$

$$dI_{scat} = \frac{\omega^2}{4\pi} |\vec{n} \times \vec{e}|^2 |f(\theta)|^2 d\Omega, \quad dI_{abs} = \frac{\omega^2}{4\pi} |\vec{n} \times \vec{e}|^2 |g(\theta)|^2 \frac{R^4}{r^2} d\Omega. \quad (34)$$

Finally for the cross sections we have expressions:

$$\frac{d\sigma_{scat}}{d\Omega} = |\vec{n} \times \vec{e}|^2 |f(\theta)|^2, \quad \frac{d\sigma_{abs}}{d\Omega'} = R^2 |\vec{n} \times \vec{e}|^2 |g(\theta)|^2. \quad (35)$$

In the last formula for the absorption cross section we used the symmetry of the system ($r' \rightarrow R^2/r$) mentioned in [14, 16] and replaced the spherical angle $d\Omega(R^2/r^2)$ into $d\Omega'$ simply because $dr' \rightarrow -(R/r)^2 dr$

We can also find the corresponding cross sections for the scalar field ϕ using the fact that $i\omega\phi = \vec{\nabla} \vec{A}$. Thus we have:

$$\frac{d\sigma_{scat}^\phi}{d\Omega} = |\vec{n} \cdot \vec{e}|^2 |f(\theta)|^2, \quad \frac{d\sigma_{abs}^\phi}{d\Omega'} = R^2 |\vec{n} \cdot \vec{e}|^2 |g(\theta)|^2. \quad (36)$$

The total cross section of electromagnetic and scalar fields is the sum $\sigma + \sigma^\phi$. As it is easy to see the total absorption and scattering cross sections do not depend on polarization vector [13] because

$$|\vec{n} \times \vec{e}|^2 + |\vec{n} \cdot \vec{e}|^2 = 1.$$

To evaluate the integral we have to calculate the vector product

$$|\vec{n} \times \vec{e}|^2 = \cos^2\theta \cos^2\varphi + \sin^2\varphi$$

therefore for the total cross section we get

$$\sigma = \frac{1}{2} \int (1 + \cos^2\theta) |f(\theta)|^2 d\Omega \quad (37)$$

Using the formulas

$$\cos^2\theta = \frac{4\pi}{3} Y_{10} Y_{10}^*$$

and

$$\sqrt{\frac{4\pi}{3}} Y_{10} * Y_{lm} = \sqrt{\frac{(l+1)^2 - m^2}{(2l+1)(2l+3)}} Y_{l+1m} + \sqrt{\frac{(l^2 - m^2)}{(2l-1)(2l+1)}} Y_{l-1m}$$

one can evaluate completely the integration in (35) and get expression for the cross section

$$\sigma_{scat} = 4\pi \sum_{l=0}^{\infty} \frac{(3l^2 + 3l - 2)}{(2l-1)(2l+3)} (2l+1) |f_l|^2 + 2\pi \sum_{l=0}^{\infty} \frac{(l+1)(l+2)}{(2l+3)} (f_l f_{l+2}^* + f_{l+2} f_l^*) \quad (38)$$

Using the expression for the amplitudes f_l we can find the lowest term in the scattering cross section

$$\sigma_1^{scat} = \frac{8\pi}{3} |f_0|^2 = \frac{8\pi}{3} R^2 (R\omega)^2. \quad (39)$$

We have to compare this cross section with the Thompson cross section

$$\sigma_T = \frac{8\pi}{3} \frac{e^4}{M_{eff}^2}$$

where M_{eff} is the effective mass of the charged object. The behaviour of these cross sections as a function of ω is essentially different and comes as a big surprise, because the cross section instead of approaching a constant tends to zero! The only way to explain this result is to analyze the effective mass of the spike

$$M_{eff}^2 = \frac{e^4}{R^2(R\omega)^2}. \quad (40)$$

We observe that it is finite for nonzero $R\omega$, demonstrating that dynamical mass involved in the process is finite and is of order of the wave length $1/\omega$. In the low energy limit it tends to infinity demonstrating that "static" mass of the spike is infinite.

In the same way one can find that the absorption cross section is

$$\sigma_1^{abs} = \frac{8\pi}{3} R^2 |g_0|^2 = \frac{8\pi}{3} R^2, \quad (41)$$

and contrary to the scattering one (39) tends to a constant! Thus we see that the extraterrestrial spike sucks almost all electromagnetic radiation and therefore acts as a black hole. This result corresponds to the dipole radiation when we consider electromagnetic perturbation propagating on the spike in the form of P-wave ($j=1$) which is coming down the spike with a polarization along a direction parallel to the brane [13].

We can also compute the corresponding cross sections for the scalar field, the only difference is in the coefficient: instead of $8\pi/3$ it is $4\pi/3$ and the total cross section $\sigma + \sigma^\phi$ comes with coefficient 4π .

5 Partial electromagnetic waves

In previous section we used partial wave expansion, but it does not correspond to the physical expansion over total angular momentum of the electromagnetic field. To do so and to distinguish corresponding partial waves with definite parity we have to expand the vector potential over the vector spherical harmonics:

$$\begin{aligned} \vec{Y}_{jm}^{el} &= \frac{1}{\sqrt{j(j+1)}} \nabla_{\vec{n}} Y_{jm}, & P &= (-1)^j; \\ \vec{Y}_{jm}^{mag} &= [\vec{n} \times \vec{Y}_{jm}^{el}], & P &= (-1)^{j+1}; \\ \vec{Y}_{jm}^{lon} &= \vec{n} Y_{jm}, & P &= (-1)^j. \end{aligned} \quad (42)$$

where $j = 1, 2, 3, \dots$ and the corresponding parity is given on the r.h.s. For any given angular momentum j there is one even and one odd parity state. The fields carrying angular momentum j and the parity $(-1)^j$ are electric multipoles Ej and the parity $(-1)^{j+1}$ are magnetic multipoles Mj .

Using this basic functions we can rederive the expansion (31) in the form

$$\vec{A}_{scat} = \frac{1}{2} \sum_{j=1, \lambda=0 \pm 1}^{\infty} \sqrt{4\pi(2l+1)} (\vec{e}^* \cdot \vec{e}_\lambda^*) \{ f_{j\lambda}^{el} \vec{Y}_{j\lambda}^{el} + f_{j\lambda}^{mag} \vec{Y}_{j\lambda}^{mag} + f_{j\lambda}^{lon} \vec{Y}_{j\lambda}^{lon} \} \frac{e^{ikr}}{r} \quad (43)$$

where $\vec{e}_0 = \vec{e}_z$, $\vec{e}_{\pm 1} = \vec{e}_x \pm \vec{e}_y$ ($\vec{e} \cdot \vec{e}_0 = 0$) and

$$\begin{aligned} f_{j\lambda}^{el} &= -\lambda \frac{j f_{j+1} + (j+1) f_{j-1}}{2j+1}, & \lambda = \pm 1; & f_{j\lambda}^{el} = \frac{\sqrt{j(j+1)}}{2j+1} (-f_{j+1} + f_{j-1}), & \lambda = 0; \\ f_{j\lambda}^{mag} &= i f_j, & \lambda = \pm 1; & f_{j\lambda}^{mag} = 0, & \lambda = 0; \\ f_{j\lambda}^{lon} &= -\lambda \frac{\sqrt{j(j+1)}}{2j+1} (-f_{j+1} + f_{j-1}), & \lambda = \pm 1; & f_{j\lambda}^{lon} = \frac{(j+1) f_{j+1} + j f_{j-1}}{2j+1}, & \lambda = 0. \end{aligned} \quad (44)$$

In the last formula the amplitudes $f_{j\lambda}^{el}, f_{j\lambda}^{mag}$ ($\lambda = \pm 1$) describe physical transverse degrees of freedom of electromagnetic field and using the fact that $i\omega\phi = \vec{\nabla} \cdot \vec{A}$ we can identify the amplitude $f_{j\lambda}^{lon}$ as describing scattering of the scalar field.

The partial cross sections can be represented now in the form:

$$\sigma_j^{scat} = 2\pi(2j+1)|f_j^{(\delta)}|^2, \quad (45)$$

where index $\delta = el, mag$ describes electric or magnetic multipoles. For the electric dipole, quadruple and higher multipoles the amplitudes are:

$$f_1^{el} = \frac{2f_0 + f_2}{3}, \quad f_2^{el} = \frac{3f_1 + 2f_3}{5}, \quad f_j^{el} \simeq \frac{j+1}{2j+1} f_{j-1}$$

and the corresponding scattering cross sections are:

$$\sigma_1^{el} = 6\pi|f_1^{el}|^2, \quad \sigma_2^{el} = 10\pi|f_2^{el}|^2, \quad \sigma_j^{el} = \frac{\pi(j+1)^2}{2(2j+1)} \left(\frac{\Gamma(-j+3/2)}{\Gamma(j+1/2)} \right)^4 R^2 (R\omega/2)^{8j-6}. \quad (46)$$

For the magnetic dipole and quadruple radiation we have

$$\sigma_1^{mag} = 6\pi|f_1|^2, \quad \sigma_2^{mag} = 10\pi|f_2|^2, \quad \sigma_j^{mag} = \frac{\pi(2j+1)}{2} \left(\frac{\Gamma(-j+1/2)}{\Gamma(j+3/2)} \right)^4 R^2 (R\omega/2)^{8j+2}. \quad (47)$$

Similar formulas are valid for the absorption amplitude $\vec{e} g(\theta) e^{ikR^2/r}$, therefore for electric multipoles we have:

$$\sigma_j^{abs} = \frac{\pi(j+1)^2}{2(2j+1)} \left(\frac{\Gamma(-j+3/2)}{\Gamma(j+1/2)} \right)^2 R^2 (R\omega/2)^{4j-4} \quad (48)$$

and one can also get less dominant magnetic multipoles.

The general asymptotic behaviour of the partial scattering cross sections have the form:

$$\sigma_j^{scat} \sim R^2 (R\omega)^{8j-6}, \quad j = 1, 2, 3, \dots \quad (49)$$

This behaviour essentially differs from the one in electrodynamics being ω^{2j-2} . The absorption cross section is:

$$\sigma_1^{abs} \sim R^2 \rightarrow Const, \quad \sigma_j^{abs} \sim R^2 (R\omega)^{4j-4}, \quad j = 2, 3, \dots \quad (50)$$

this behaviour is appealing for the string interpretation of the spike soliton because it is a constant for $j = 1$ and tends to zero for higher $j > 1$, therefore the propagation of $j = 1$ mode is indeed distinctive.

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6 Appendix

In section 3 the amplitudes of scattering and absorption have been calculated by solving equation (19) in lowest order in $\lambda = \frac{1}{2}R\omega$ by standard continuity argument. The known mathematics of Mathieu's equation [26, 27, 28] allows to calculate systematic expansion in all orders in λ . The formulae needed for our purpose have been summarized in [29].

The transition (T_ℓ) and reflection (R_ℓ) partial amplitudes are (instead of equations (27), (28))

$$T_\ell = -2i\chi_0^{-1} \frac{\frac{1}{2}[1 + (1 + \sum_1^\infty \lambda^{4k} \bar{\mu}_k)^{-2}](1 + \sum_1^\infty \lambda^{4k} \chi_k)^{-1}}{[1 + \chi_0^{-2}(1 + \sum_1^\infty \lambda^{4k} \chi_k)^{-2}(1 + \sum_1^\infty \lambda^{4k} \bar{\mu}_k)^{-2}]}, \quad (51)$$

$$R_\ell = (-1)^\ell \frac{[1 - \chi_0^{-2}(1 + \sum_1^\infty \lambda^{4k} \chi_k)^{-2}](1 + \sum_1^\infty \lambda^{4k} \bar{\mu}_k)^{-1}}{[1 + \chi_0^{-2}(1 + \sum_1^\infty \lambda^{4k} \chi_k)^{-2}(1 + \sum_1^\infty \lambda^{4k} \bar{\mu}_k)^{-2}]}. \quad (52)$$

We have evaluated the first correction $\mathcal{O}(\lambda^4)$ only:

$$\begin{aligned} \chi_0 &= \lambda^{-(2\ell+1)} \frac{\Gamma(\ell + \frac{3}{2})}{\Gamma(\frac{1}{2} - \ell)}, \quad \mu_0 = \frac{1}{2}(\ell + \frac{1}{2}) \\ \mu_1 &= \frac{1}{\ell + \frac{1}{2}} \left(\frac{1}{\ell + \frac{3}{2}} - \frac{1}{\ell - \frac{1}{2}} \right), \quad \bar{\mu}_1 = 2\pi i \mu_1 \\ \chi_1 &= 2\mu_1 \left[\psi(\ell + \frac{3}{2}) + \psi(\frac{1}{2} - \ell) - 2 \ln \lambda \right] - \frac{1}{(\ell + \frac{3}{2})^2} - \frac{1}{(\ell - \frac{1}{2})^2}. \end{aligned} \quad (53)$$

The first order correction changes the leading small $R\omega$ behaviour of the scattering amplitude $R_\ell - 1$,

$$R_\ell = 1 - 8\lambda^2 \delta_{\ell,0} + \lambda^4 (2\pi i \mu_1(\ell) + 32 \delta_{\ell,0}) + \mathcal{O}(\lambda^8, \lambda^{4\ell+2}) \quad (54)$$

This implies that all partial scattering cross sections for $j \geq 1$ behave like

$$\sigma_j^{scatt} \sim R^2 (R\omega)^6 \quad (55)$$

with a coefficient decreasing at large j like j^{-5} . This is an essential change compared to the one anticipated from the zero approximation, $\sigma_j^{scatt} \sim R^2 (R\omega)^{8j-6}$.

The corrections to the transition amplitude are less significant

$$T_\ell = 2\lambda^{2\ell+1} \frac{\Gamma(\frac{1}{2} - \ell)}{\Gamma(\ell + \frac{3}{2})} (1 - 4\lambda^2 \delta_{\ell,0} - \lambda^4 (2\pi i \mu_1(\ell) + \chi_1(\ell)) + \mathcal{O}(\lambda^8, \lambda^{4\ell+2})). \quad (56)$$

The leading behaviour for $R\omega \rightarrow 0$ is really the one expected from zero approximation.

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